

# Parity-violating asymmetry of $W$ bosons produced in $p$ - $p$ collisions

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## Abstract

The parity-violating asymmetry is an ideal tool to study the quark helicity distribution in the proton. We study the parity-violating asymmetry of  $W^\pm$  bosons produced by longitudinally polarized  $p$ - $p$  collision in RHIC, based on predictions of quark distributions of the proton in the SU(6) quark-spectator-diquark model and a perturbative QCD based counting rule analysis. We find that the two models give nearly equal asymmetry for  $W^+$  but that for  $W^-$  quite different. Therefore future experiments on such quantity can help to clarify different predictions of the value  $\Delta d(x)/d(x)$  at  $x \rightarrow 1$  in the proton.

*Key words:* quark helicity distribution, nucleon-nucleon collisions, phenomenological quark models, W bosons

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## 1 Introduction

The helicity distribution is one of the three fundamental quark distributions of the nucleon, and it has been explored for more than two decades in deep inelastic scattering progresses. Deep inelastic scattering (DIS) experiments of polarized electrons and muons from polarized nucleons have confirmed us that the total quark-plus-antiquark helicity sum is only about 1/4 to 1/3 of the proton spin [1,2,3,4,5]. However, inclusive DIS experiments cannot provide

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information on the polarized quark and antiquark density separately. Semi-inclusive DIS measurements are one of the approaches to achieve a separation of quark and antiquark helicity densities [6,7]. But these experiments depend on the details of the fragmentation process, so their accuracy is limited. In these experiments, spin asymmetry is one of the most important quantity to study the quark helicity distributions. The Jefferson Lab Hall A Collaboration has released a precision result on the neutron spin asymmetry [8,9] at large  $x$ , which provides an examination on some known quark distribution models. The disadvantage of semi-inclusive DIS process can be avoided by using the production of  $W$  bosons at RHIC [10,11].

About ten years ago, Soffer and his collaborators pointed out that the polarization of  $u$ ,  $d$ ,  $\bar{u}$ , and  $\bar{d}$  in the proton will be measured directly and precisely using the maximum parity violation of  $W$  bosons in  $u\bar{d} \rightarrow W^+$  and  $d\bar{u} \rightarrow W^-$  at high  $Q^2$  [14,15]. At leading order, the single spin asymmetry, or the parity-violating asymmetry, of  $W^\pm$  will approach to the ratio  $\Delta q(x)/q(x)$  when the absolute value of the rapidity of  $W$ ,  $y_W$ , is large. Some works have been done to calculate this asymmetry with different quark distribution models [12,13,14,15,16,17,18,19,20]. The purpose of this work is to show that the parity-violating asymmetry of  $W^-$  bosons in longitudinally polarized  $p$ - $p$  process at RHIC can provide a crucial test of different predictions on  $\Delta d(x)/d(x)$  from a pQCD based analysis and the SU(6) quark-spectator-diquark models, because it is very sensitive to the value of  $\Delta d(x)/d(x)$  at  $x \rightarrow 1$  in the proton.

The parity-violating asymmetry is defined as the difference of the cross section of the left-handed ( $-$ ) beams and that of the right-handed ( $+$ ) beams, divided by the sum of them, or using the number density of the beams instead and normalized by the polarization:

$$A_L = -\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = -\frac{1}{P} \times \frac{N'_+ - N'_-}{N'_+ + N'_-}. \quad (1)$$

The production of  $W$  is dominated by  $u$ ,  $d$ ,  $\bar{u}$ , and  $\bar{d}$  contributions, with some contamination of  $s$ ,  $c$ ,  $\bar{s}$ , and  $\bar{c}$  contributions, mostly through quark mixing. Therefore  $W$  production is an ideal tool to study the spin-flavor structure of the nucleon. The expected sensitivities at RHIC are also given in Ref. [10]. PHENIX and STAR both expect to get the enough  $W^+(W^-)$  data for analysis.

At RHIC, the  $W^\pm$  will be produced in sub-processes  $u\bar{d} \rightarrow W^+$  and  $d\bar{u} \rightarrow W^-$  by colliding beams of protons spinning alternately left- and right-handed.

For Standard Model, the leading order production of  $W^+$ s, is partly illustrated in Fig.1 [10]. Due to that the production of left-handed  $W$  bosons violate parity, the asymmetry can be constructed from polarized beams with two directions. As Fig. 1 shows, the  $u$  quark only comes from the polarized proton,

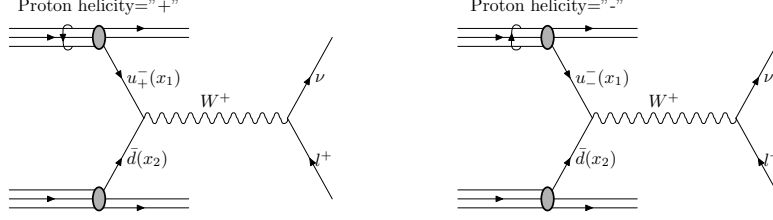


Fig. 1. A possible production of  $W^+$  in  $p$ - $p$  collision, at lowest order. The  $u$  quark comes from the polarized proton and the  $\bar{d}$  quark comes from the unpolarized proton. the asymmetry should be expressed as:

$$A_L^{W^+} = \frac{u_-(x_1)\bar{d}(x_2) - u_+(x_1)\bar{d}(x_2)}{u_-(x_1)\bar{d}(x_2) + u_+(x_1)\bar{d}(x_2)} = \frac{\Delta u(x_1)}{u(x_1)}. \quad (2)$$

In the above equation, superscripts refer to the quark helicity and subscripts refer to that of proton.  $q(x_n)$  is short for  $q(x_n, Q^2)$ , where  $q = u, d$  and  $n = 1, 2$ ,  $Q^2 = M_W^2$  and  $M_W$  is the mass of  $W$  bosons.

For the situation that the  $\bar{d}$  quark comes from the polarized proton, the asymmetry should be simply written as:

$$A_L^{W^+} = \frac{\bar{d}_-(x_1)u(x_2) - \bar{d}_+(x_1)u(x_2)}{\bar{d}_-(x_1)u(x_2) + \bar{d}_+(x_1)u(x_2)} = -\frac{\Delta \bar{d}(x_1)}{\bar{d}(x_1)}. \quad (3)$$

In general, the asymmetry is a superposition of the two cases:

$$A_L^{W^+} = \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta \bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}. \quad (4)$$

The asymmetry of  $W^-$  can be got easily by interchanging  $u$  and  $d$ :

$$A_L^{W^-} = \frac{\Delta d(x_1)\bar{u}(x_2) - \Delta \bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)}. \quad (5)$$

Higher-order corrections change the asymmetries only a little [16,17].

The kinematics of  $W$  production and Drell-Yan production of lepton pairs are the same. The momentum fraction carried by the quarks and antiquarks,  $x_n (n = 1, 2)$ , can be determined by the rapidity of  $W$ ,  $y_W$ ,

$$x_1 = \frac{M_W}{\sqrt{s}} e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}} e^{-y_W}, \quad (6)$$

where  $\sqrt{s}$  is the center-of-mass energy. This picture is valid only for the predominant production of  $W$ s at  $p_T = 0$ . In reality, the  $W$  has  $p_T$ , which comes

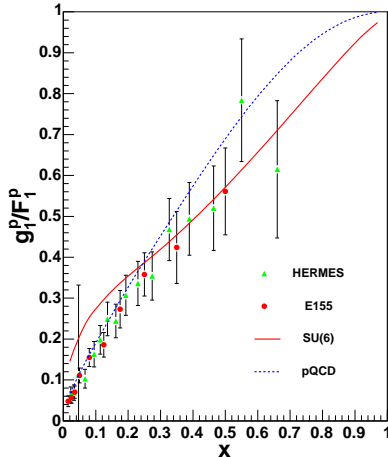


Fig. 2. Predictions on  $g_1^p/F_1^p$  of quark-diquark model [23] and pQCD based analysis [26] for  $Q^2 = 4\text{GeV}^2$ , and are compared to the experimental data from HERMES [5] and E155 [3].

from the higher level contributions such as  $gu \rightarrow W^+d$  and  $ud \rightarrow W^+g$ , or the primordial  $p_T$  for the initial parton. But we must point out that when  $y_W \gg 1$ ,  $A_L^{W^+} \approx \Delta u(x)/u(x)$ , and  $A_L^{W^-} \approx \Delta d(x)/d(x)$ . So if we take  $p_T = 0$  in calculation, the results are also valuable at large  $y_W$ .

The experimental difficulty is that the  $W$  is observed through its leptonic decay  $W \rightarrow l\nu$ , and only the charged lepton is observed. The connection of the rapidity of observed leptons and that of  $W$ s has been described in detail in Ref. [10]. And the expected cross section of  $W$ s has also been estimated in that article.

## 2 Quark Distributions

From above equations, we can calculate the asymmetries of  $W^\pm$ s if the quark helicity distributions are given. Two known models, the quark-diquark model [21,22,23] and a pQCD based counting rule analysis [24,25,26,27,28], can provide predictions of the quark distributions. Both models have their advantages and have played important roles in the investigation of various nucleon structure functions. However, there are still some unknowns concerning the sea content of the nucleon and the large  $x$  behaviors of valence quark. For example, there are still some uncertainties concerning the flavor decomposition of the quark helicity distributions at large  $x$ , especially for the less dominant  $d$  valence quark of the proton. There are two different theoretical predictions of the ratio  $\Delta d(x)/d(x)$  at  $x \rightarrow 1$ : the pQCD base counting rule analysis [26] predicts  $\Delta d(x)/d(x) \rightarrow 1$  whereas the SU(6) quark-diquark model [23] predicts  $\Delta d(x)/d(x) \rightarrow -1/3$ . Experimental data of recent years show that both

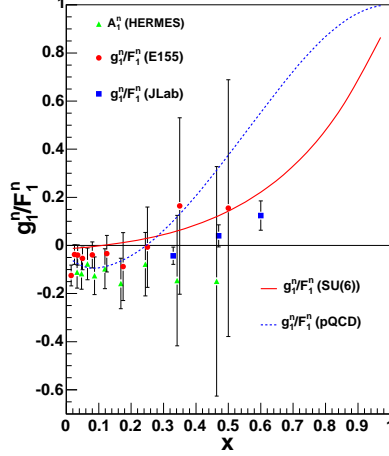


Fig. 3. Predictions on  $g_1^n/F_1^n$  of quark-diquark model [23] and pQCD based analysis [26] for  $Q^2 = 4 \text{ GeV}^2$ , and are compared to experimental data from JLab [8,9] and HERMES [4].

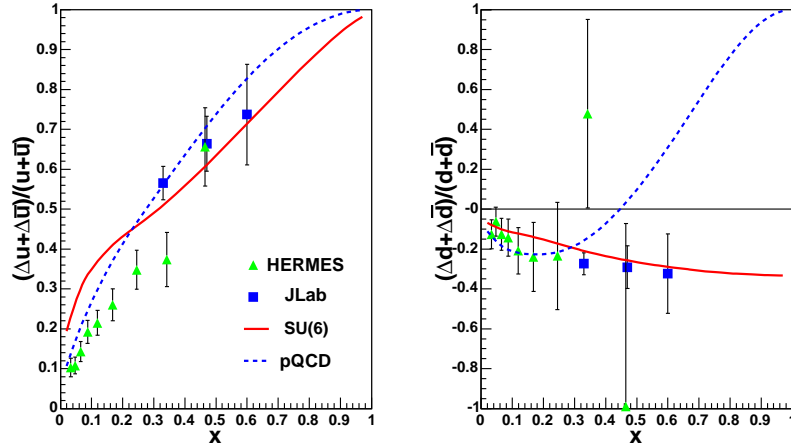


Fig. 4. Predictions on the flavor decomposition  $(\Delta u + \Delta \bar{u})/(u + \bar{u})$  and  $(\Delta d + \Delta \bar{d})/(d + \bar{d})$  of quark-diquark model [23] and pQCD based analysis [26] for  $Q^2 = 4 \text{ GeV}^2$ , and are compared to experimental data from JLab [8,9] and HERMES [7].

of these two models are good in the predictions of  $g_1^p$ , and of the  $\Delta u(x)/u(x)$  extracted from it. But results of Jefferson Lab Hall A Collaboration show that the pQCD prediction on  $A_1^n$  deviates from the data [8,9]. Figures 2,3 and 4 show the predictions of these two models and the comparison with experimental data. It seems that the pQCD predictions fit the data good at small  $x < 0.3$  whereas the SU(6) quark-diquark model works good at large  $x > 0.3$ .

We can expect that the different predictions on  $\Delta d(x)/d(x)$  of the two models may lead to prominent differences in calculating the parity-violating asymmetry of  $W^-$ , due to the case that  $A_L^{W^-} \approx \Delta d(x)/d(x)$  when  $y_W$  is large.

Detailed constructions of the quark helicity distributions in the two models can be found in Ref. [29].

To make realistic predictions of measurable quantities, we need also to take into the sea quark contribution in the two model constructions. In the quark-diquark case, this can be achieved by adopting one set of unpolarized quark distribution parametrization as input, and then use theoretical relations to connect the quark helicity distributions with the unpolarized distributions [23,30]:

$$\begin{aligned}\Delta u_v(x) &= [u_v(x) - \frac{1}{2}d_v(x)]W_S(x) - \frac{1}{6}d_v(x)W_V(x), \\ \Delta d_v(x) &= -\frac{1}{3}d_v(x)W_V(x).\end{aligned}\tag{7}$$

$W_S(x)$  and  $W_V(x)$  are the Melosh-Wigner rotation factors [31] for spectator scalar and vector diquarks. We use the valence quark momentum distributions  $u_v(x)$  and  $d_v(x)$  from quark distribution parametrization, but with  $W_S(x)$  and  $W_V(x)$  from the model calculation [30]. In this way we can take into account the sea contribution for the unpolarized quark distribution from the input parametrization. The energy scale dependence and the  $Q^2$  evolution behaviors of the quark distributions can be reflected by the explicitly  $Q^2$  dependence of the input quark distribution parametrization. This can provide a more reliable prediction for the magnitude and shape of a physical quantity than directly from the model calculation. CTEQ DIS [32] (CTEQ5 set 2) quark distributions are used for parametrization to investigate the evolution of the helicity distribution. From Fig. 5 we found that the ratios of  $\frac{\Delta u(x) + \Delta \bar{u}(x)}{u(x) + \bar{u}(x)}$  and  $\frac{\Delta d(x) + \Delta \bar{d}(x)}{d(x) + \bar{d}(x)}$  changed little with the increase of  $Q^2$  in this model.

For the pQCD based analysis, we take the same consideration as the above and make the following connection to relate the pQCD model quark distributions with the parametrization:

$$u_v^{\text{pQCD}}(x) = u_v^{\text{para}}(x),\tag{8}$$

$$d_v^{\text{pQCD}}(x) = \frac{d_v^{\text{th}}(x)}{u_v^{\text{th}}(x)} u_v^{\text{para}}(x),\tag{9}$$

$$\Delta u_v^{\text{pQCD}}(x) = \frac{\Delta u_v^{\text{th}}(x)}{u_v^{\text{th}}(x)} u_v^{\text{para}}(x),\tag{10}$$

$$\Delta d_v^{\text{pQCD}}(x) = \frac{\Delta d_v^{\text{th}}(x)}{u_v^{\text{th}}(x)} u_v^{\text{para}}(x),\tag{11}$$

where the superscripts “th” means the pure theoretical calculation in the pQCD analysis, and “para” means the input from parametrization. The su-

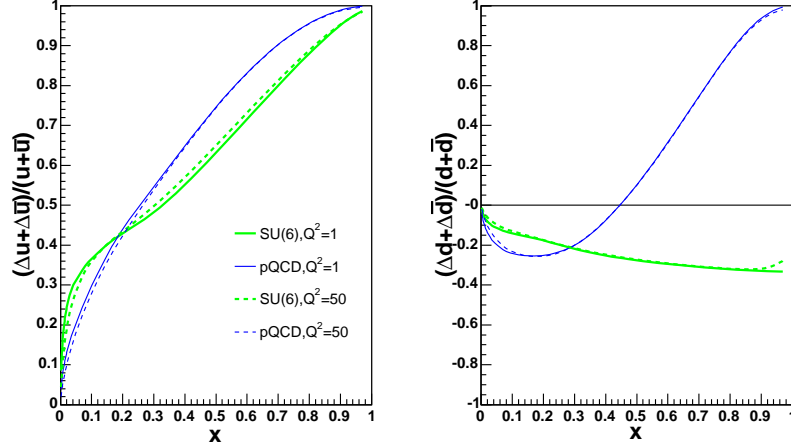


Fig. 5. The  $Q^2$  evolution of the helicity distributions. Prediction of the decomposition  $(\Delta u + \Delta \bar{u})/(u + \bar{u})$  and  $(\Delta d + \Delta \bar{d})/(d + \bar{d})$  of quark-diquark model (thick curves) and pQCD based analysis (thin curves) for  $Q^2 = 1 \text{ GeV}^2$  (solid curves) and  $50 \text{ GeV}^2$  (dashed curves).

perscript “pQCD” means that the new quark distributions keep exactly the same flavor and spin structure as those in the pQCD analysis, but with detailed  $x$ -dependent behaviors more close to the realistic situation. This is equivalent to using a factor,  $u_v^{\text{para}}(x)/u_v^{\text{th}}$ , to adjust each pure *theoretically* calculated quantity to a more realistic pQCD *model* quantity. In this way we can take into account the sea contribution by using the sea quark distributions from parametrization, while still keep the pQCD model behaviors of the valence quark distributions. We also use CTEQ DIS quark distribution functions as input to investigate the  $Q^2$  evolution of helicity distributions. The results are drawn in Fig. 5. We found the ratios changed little with the increase of  $Q^2$  in this model as those in SU(6) quark-diquark model.

Thus we have two set of quark distributions of  $\Delta q(x)$  and  $q(x)$ , which keeps the same valence behaviors as in the quark diquark model and pQCD based analysis prediction. In the SU(6) model, the sea quarks are unpolarized, and only valence quarks are polarized. Both of these two models do not give the polarized distribution of the sea. HERMES data [33] show that the absolute value of polarized sea quark distributions is much smaller than the absolute value of polarized valence quark distributions when  $x > 0.1$ . So it does not matter for large  $x$  and  $y_W$  if we omit terms including  $\Delta \bar{q}(x)$  in the calculation. The  $Q^2$  evolution behavior of the decomposition indicates that the asymmetry changes little when calculated with distribution functions at different  $Q^2$ , thus the results and conclusion of this work will not influenced qualitatively by the  $Q^2$  evolution effect.

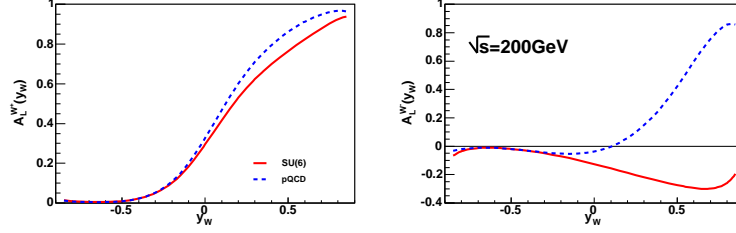


Fig. 6. The value of  $A_L^W(y_W)$  in different models. The solid curves correspond to the SU(6) quark-spectator-diquark model, and the dashed curves corresponds to the pQCD based analysis, for  $\sqrt{s} = 200$  GeV.

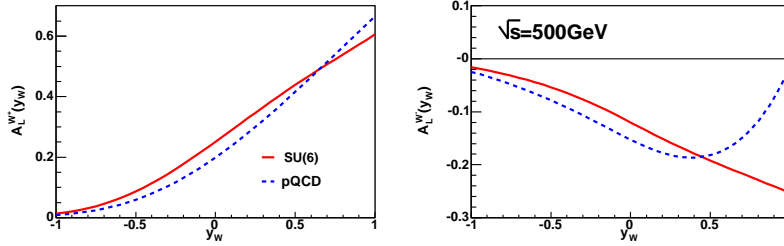


Fig. 7. The value of  $A_L^W(y_W)$  in different models. The solid curves correspond to the SU(6) quark-spectator-diquark model, and the dashed curves correspond to the pQCD based analysis, for  $\sqrt{s} = 500$  GeV.

### 3 Numerical Calculations

With the definition of  $A_L$  and the quark distributions, we can perform numerical calculations using different models. For the unpolarized quark distributions, we use the CTEQ LO parametrization [32] (CTEQ5 set 3) as input for both the SU(6) quark-spectator-diquark model and pQCD based analysis distribution functions described above.  $Q = M_W = 80.419$  GeV is taken to get quark distributions from the CTEQ LO parametrization, where  $M_W$  is the mass of  $W$  bosons.

The polarized  $p$ - $p$  collisions at RHIC will take place at center-of-mass energies of  $\sqrt{s} = 200 \sim 500$  GeV. So we calculate  $A_L^W$  at  $\sqrt{s} = 200$  GeV and  $\sqrt{s} = 500$  GeV. Although the production of  $W$  at 200 GeV is not feasible with the designed luminosity at RHIC, we provide the calculation as a reference to show the model dependent predictions at different collision energies. Results of  $A_L^{W^\pm}(y_W)$  are shown in Fig. 6 and Fig. 7.

It is clear that  $A_L^{W^+}$  are nearly equal in these two models, but  $A_L^{W^-}$  show great difference between the two models at both the two energy scales.

As we mentioned before,  $A_L^{W^-} \approx \Delta d(x)/d(x)$  when  $y_W$  is large, which means that  $x_1$  is large. We may see the difference more clearly by taking  $x_1$  as the independent variable. To examine this, we calculate  $A_L^W$  again. Results are



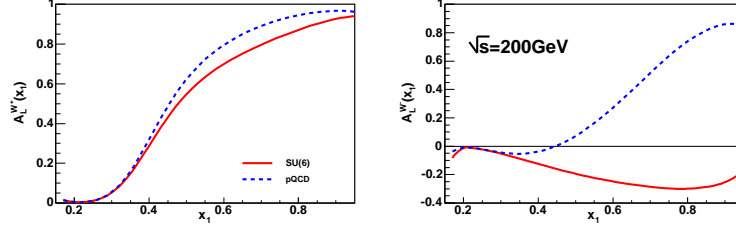


Fig. 8. The value of  $A_L^W(x_1)$  in different models. The solid curves correspond to the SU(6) quark-spectator-diquark model, and the dashed curves correspond to the pQCD based analysis, for  $\sqrt{s} = 200$  GeV.

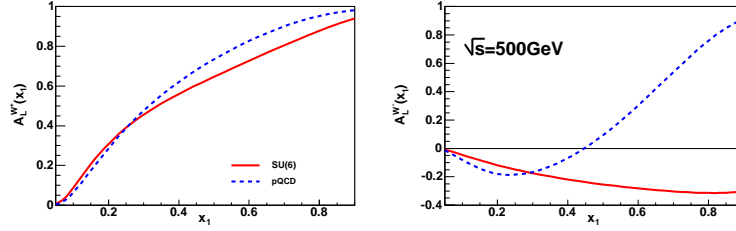


Fig. 9. The value of  $A_L^W(x_1)$  in different models. The solid curves correspond to the SU(6) quark-spectator-diquark model, and the dashed curves correspond to the pQCD based analysis, for  $\sqrt{s} = 500$  GeV.

shown in Fig. 8, where  $\sqrt{s} = 200$  GeV, and Fig. 9, where  $\sqrt{s} = 500$  GeV.

The results convince us that the difference of  $A_L^{W^-}$  in these two models comes from the different predictions on  $\Delta d(x)/d(x)$  of the proton. We expect that experiments at RHIC can give an examination.

## 4 Summary

The parity-violating asymmetry in the production of  $W$  bosons in longitudinally polarized  $p$ - $p$  process can be measured at RHIC. It is sensitive to quark distributions of the proton. A pQCD based analysis and the SU(6) quark-spectator-diquark model give obvious different results on the parity-violating asymmetry of  $W^-$ , which is sensitive to the value of  $\Delta d(x)/d(x)$ , a basic quantity which is difficult to be measured in other processes. Therefore RHIC experiments results can give a powerful examination on different predictions and enrich our knowledge on the spin structure of the proton.

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